This thesis investigates the fractional Stefan problem(FSP) governed by the space-fractional diffusion–advection equation under specific boundary conditions. The main objective is to develop analytical and semi-analytical approaches that accurately describe anomalous diffusion and phase transition phenomena in materials with nonlocal and memory-dependent thermal behavior, which are inadequately represented by classical models.

In the first part, the study formulates and analyzes the space-fractional Stefan problem using Caputo-type spatial fractional derivatives of the model the nonlocal nature of heat transfer in heterogeneous and complex media. By employing a similarity transformation, the governing space-fractional partial differential equation (PDE) is reduced to an equivalent fractional ordinary differential equation (ODE).The Laplace–Adomian Decomposition Method (LADM) is then utilized to obtain a series solution expressed through the three-parameter Mittag-Leffler function, which effectively captures the memory effects and spatial correlations intrinsic to fractional diffusion. The obtained results demonstrate that the fractional solutions converge to their classical counterparts as the fractional order approaches unity, confirming the validity and consistency of the proposed formulation.

In the second part, the research extends the analysis to the space-fractional diffusion–convection equation to explore self-similar solutions within the framework of the fractional Stefan problem. Through rescaling techniques and a comparison theorem, the existence, structure, and qualitative behavior of self-similar solutions are established. The study focuses on key properties such as nonnegativity, regularity, and the geometric characteristics of the similarity profiles.The results reveal the significant impact of the fractional order on front propagation and anomalous diffusion dynamics, offering deeper insights into heat and mass transfer processes in complex and heterogeneous media. Overall, this work contributes to the advancement of fractional modeling of phase-change phenomena, demonstrating that fractional calculus provides a powerful mathematical framework for describing nonlocality, memory, and anomalous transport behaviors in real-world systems.